

# SEARCH FOR SOLITONS IN TWO-HIGGS EXTENSIONS OF THE STANDARD MODEL \*

T. N. TOMARAS <sup>†</sup>

*Department of Physics, University of Crete, and Research Center of Crete*

*714 09 Heraklion, Crete, Greece*

E-mail: tomaras@plato.iesl.forth.gr

## ABSTRACT

We report on the status of our search for quasi-topological solitons of various dimensions in realistic field theoretical models of condensed matter and of elementary particle physics.

## 1. Introduction

The search for stable lumps in the Weinberg-Salam model has a long history. It has revealed a rich structure of classical solutions including the sphaleron<sup>1,2,3,4</sup>, deformed sphalerons<sup>5,6,7,8</sup> and vortex strings<sup>9,10,11,12,13</sup>. Such solutions could play a role in understanding (B+L)-violation and structure formation in the early universe, but they are all classically-unstable or/and extended. They have therefore no direct present-day manifestation, contrary to long-lived particles whose relic density could at least in principle be detected.

The existence of particle-like excitations has, on the other hand, been argued for in the context of a strongly- interacting Higgs sector<sup>14,15,16,17,18</sup>. The advocated particles can be thought of as technibaryons of an underlying technicolor model. They are described in bosonic language by winding solitons of an effective non-renormalizable lagrangian for the pseudo-goldstone-boson (or technipion) field, much like skyrmions<sup>19</sup> of the effective chiral lagrangian of *QCD*. This is of course a phenomenological description, since the properties of such hypothetical particles cannot be calculated reliably within a semi-classical expansion. Furthermore, in view of the difficulties facing technicolor models, the possibility of a strongly-interacting Higgs sector is not theoretically appealing.

It would be clearly more interesting if classically-stable winding excitations could arise in a *weakly-coupled* scalar sector. To be more precise let us decompose the Higgs-doublet field into a real (positive) magnitude and a group-phase:  $\Phi = FU$ , and consider static configurations with  $U(x)$  wrapping  $N$  times around the  $SU(2)$  manifold. These are potentially unstable for at least three distinct reasons: (a) because  $N$  is not conserved whenever the magnitude  $F$  goes through zero; (b) because  $N$  is not gauge-invariant and can, in particular, be non-vanishing even in a vacuum state; and (c) because scalar-field configurations can loose their energy by shrinking to zero size<sup>20</sup>. We refer to these for short as the *radial*, *gauge* and *scale* instabilities. They can be eliminated formally by (a) taking the physical-Higgs mass  $m_H \rightarrow \infty$ , (b) decoupling the electroweak gauge fields, and (c) adding appropriate higher-derivative

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terms to the action. The question is whether classical stability can be maintained while relaxing the above conditions. This has been investigated numerically in the *minimal* case of one doublet: although one may indeed relax both the weak gauge coupling<sup>16,5</sup> and the Higgs mass<sup>21</sup> up to some finite critical values, stability cannot apparently be achieved without the non-renormalizable higher-derivative terms in the action. On the other hand, as we have demonstrated recently, metastable winding solitons do arise in renormalizable models in two<sup>22</sup> and three<sup>23</sup> space-time dimensions. The way this happens is the content of section 2. It is we believe instructive and could guide the search for lower dimensional structures in many physical systems as well as for similar semi-classical solitons in four dimensions. Section 3 contains a brief presentation of the status of our search for particle-like solutions in the context of two-Higgs extensions of the standard model. We close with some comments in the discussion section.

## 2. Quasi-topological solitons in lower dimensions

The simplest context in which the *radial* instability is an issue is a two-dimensional model of a complex scalar field with mexican-hat potential:  $V = \frac{1}{4}\lambda(\Phi^*\Phi - v^2)^2$ . The simplest way to describe in detail the winding solitons is to take space to be periodic with period  $L$ <sup>a</sup>. The condition for classical stability can in this case be derived analytically and reads<sup>22</sup>:  $m_H L > \sqrt{5}$ , where  $m_H = \sqrt{2\lambda}v$ . The classically-relevant parameter is thus the radial-Higgs mass in units of the soliton size. This follows also by comparing the loss in potential energy to the gain in gradient energy when trying to undo the winding by reducing the magnitude of the scalar. Note that the loop-expansion parameter  $\lambda L^2$ , can be taken to zero independently so as to reach a semiclassical limit.

The above winding solitons become unstable classically if we gauge the  $U(1)$  symmetry of the model. The *gauge* instability is in fact more severe than in four dimensions, because no energetic barrier opposes the turning-on of a static space-like gauge field, which is necessary to reach a winding-vacuum state. The minimal Abelian-Higgs model has thus only unstable (sphaleron) solutions<sup>25</sup><sup>b</sup>. The situation changes, however, drastically if there are more than one complex scalars. The gauge-invariant relative phases of any two of them cannot in this case wind around non-trivially in a vacuum state. An explicit analysis of this extended abelian-Higgs model<sup>22</sup> shows that winding solitons persist down to scalar masses close to the inverse soliton size: gauging and the extra Higgs enhance the stability region found in the global model.

The *scale* instability becomes an issue for the first time in three space-time dimensions. To be more precise we consider a real-triplet scalar field  $\Phi_a(x)$  ( $a = 1, 2, 3$ ) with mexican-hat potential:  $V = \frac{1}{4}\lambda(\Phi_a\Phi_a - v^2)^2$ . The limit  $\lambda \rightarrow \infty$  corresponds to the  $O(3)$  non-linear  $\sigma$ -model. This is known to possess winding solitons, characterized by non-trivial mappings of the two-sphere onto itself, and having arbitrary size<sup>26</sup>. For finite  $\lambda$  on the other hand, or in the presence of a symmetry-breaking potential, Derrick's scaling argument<sup>20</sup> shows that these solitons are unstable to shrinking. One can of course again invoke higher-derivative terms to stabilize the scale<sup>27</sup>. The

<sup>a</sup>Alternatively we may add a mass term:  $\delta V = -\mu^2 v Re(\Phi)$ , that lifts the vacuum degeneracy. Stable winding excitations, which reduce to the sine-Gordon solitons in the  $\lambda \rightarrow \infty$  limit, can be shown<sup>24</sup> numerically to exist for  $\lambda v^2/\mu > 18.8$ .

<sup>b</sup>It was claimed erroneously in [22] that it has no static solutions whatsoever. This is only correct in the  $\lambda \rightarrow \infty$  limit.

same result is however in this case achieved by a massive  $U(1)$  gauge field with only renormalizable couplings<sup>23</sup>. This can be established by perturbing around the  $O(3)$  non-linear  $\sigma$ -model limit, or else by solving numerically the equations of motion.

What do these lower-dimensional solitons teach us? *First*, they suggest by analogy that classically-stable winding solitons may exist in a weakly-coupled two-Higgs extension of the standard model. The status of our search for such localized solutions is the content of the following section.

*Second*, they are interesting in their own right, since they correspond to a new class of wall and string defects in realistic four-dimensional models of particle and condensed-matter physics. One such example are the metastable *membranes*, discovered recently<sup>28,29</sup> in two or more Higgs-doublet extensions of the standard model. These are static wall-type solutions, non-topological but classically stable in a wide region of the Higgs sector parameter space compatible with perturbative unitarity and with present phenomenological bounds. They are embeddings of the above discussed<sup>22</sup> solutions of the 2d Abelian-Higgs model with two or more complex scalars. They are characterized by the non-trivial winding of the relative  $U(1)$  phase of the neutral components of any two Higgs doublets of the extended standard model in the direction  $x$  normal to the wall. They have no electromagnetic coupling, their size is a few times the inverse of the mass  $m_A$  of the CP-odd scalar  $A^0$  (in the standard notation of the generic two-Higgs model<sup>30</sup>), while their energy per unit area is in terms of the  $A^0$  and  $W$  masses and the fine structure constant of order  $m_W^2 m_A / \alpha$ . Assuming a  $m_A \simeq 50 \text{ GeV}$  i.e. not much larger than the present experimental lower bound<sup>30</sup>, the mass of the wall is of  $\mathcal{O}(10^{10} \text{ gr/cm}^2)$ . Thus, a single wall crossing the entire Universe today would by far overclose it and can be excluded. Smaller membranes though, which either collapsed or were torn apart by quantum tunneling may have acted as seeds for the formation of galaxies. In fact, the mass of a typical galaxy is comparable to that of a membrane a few light years in size. In any case, the production and decay rates of these objects and their cosmological role has to be studied in detail, since most likely they are going to lead to firm constraints on the Higgs mass spectrum of extensions of the standard model<sup>28</sup>.

As another example, we would like to mention the static wall-type metastable solutions of the easy-plane ferromagnetic continuum in the presence of an external in-plane magnetic field  $h$ <sup>c</sup>. The dynamics of the system is described by the Landau-Lifchitz equations with energy density given in terms of the unit magnetization vector  $n_a$ ,  $a = 1, 2, 3$ , by:  $\mathcal{E} = \frac{1}{2}(\partial_i n_a)^2 + g n_3^2 + h(1 - n_1)$ . The anisotropy constant  $g$  and the external field  $h$  are both taken to be positive. The system has the unique semiclassical ground state:  $(n_1 = 1, n_2 = 0 = n_3)$  and no topologically stable domain walls. Nevertheless, it possesses a variety of wall-type static finite energy (per unit area) solutions. A particularly interesting one is  $(n_1 = \cos\Theta(x), n_2 = \sin\Theta(x), n_3 = 0)$ , with  $\Theta(x)$  satisfying  $\Theta''(x) = h \sin\Theta(x)$  and the boundary conditions  $\Theta(-\infty) =$

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<sup>c</sup>The search for finite-energy soliton solutions in ferromagnetic and antiferromagnetic systems and the study of their stability, reported briefly in this paragraph, was carried out in collaboration with Prof. N. Papanicolaou. I would also like to thank Dr. P. Tinyakov for helpful discussions about sphaleron solutions in relativistic anisotropic non-linear  $\sigma$ -models in 1+1-dimensions.

$0, \Theta(+\infty) = 2\pi$ . It can be verified that this solution is dynamically stable for  $g/h \geq 3$ .

### 3. Localized solutions in 3+1 dimensions <sup>d</sup>

We now turn our attention to the possibility of classically stable localized particle-like winding solitons in 3+1 dimensions. In the context of a two-Higgs extension of the standard model these hypothetical solitons would: (b) be characterized by the non-trivial winding of the relative phase of the two doublets, and thus be immune to the gauge mode of decay; (c) have a scale stabilized by electroweak magnetic fields and hence of order  $1/m_W$ ; and (a) hopefully stay stable for Higgs masses near  $m_W$  and thus compatible with perturbative unitarity <sup>e</sup>. Mathematically the situation is the same as in the hidden-gauge-boson models<sup>32,33</sup> of strong and electroweak interactions, except that the role of the hidden gauge bosons is here played by  $W^\pm$  and  $Z$  themselves.

We carried out a numerical search<sup>32,34,31</sup> for such stable spherically symmetric particle-like soliton solutions in a simplified version of the two-Higgs  $SU(2) \times U(1)$  model. The presumably inessential simplification consists of taking (a) the  $U(1)$  gauge coupling  $g' = 0$  as required by spherical symmetry and (b) the Higgs potential to be the sum of two "mexican hats" one for each Higgs doublet. The resulting action is:

$$S = \frac{1}{g^2} \int d^4x \left( -\frac{1}{2} \text{Tr}(W_{\mu\nu} W^{\mu\nu}) + \sum_{I=1,2} (D_\mu H_I)^\dagger (D^\mu H_I) + \sum_{I=1,2} \frac{\lambda_I}{g^2} (H_I^\dagger H_I - g^2 v_I^2)^2 \right) \quad (1)$$

with  $D_\mu H_I = (\partial_\mu + W_\mu) H_I$ ,  $I = 1, 2$  the covariant derivative on the Higgs doublets.  $W_\mu \equiv \frac{1}{2i} \tau^a W_\mu^a$  and  $W_{\mu\nu}$  is the  $SU(2)$  field strength. We used the spherically symmetric ansatz

$$\begin{aligned} W_0 &= a_0 \tau_i n_i / 2i, \\ W_i &= [(\alpha - 1) \epsilon_{ijk} \tau_j n_k / r + \beta (\delta_{ij} - n_i n_j) \tau_j / r + a_1 n_i n_j \tau_j] / 2i \\ H_I &= (\mu_I + i \nu_I n_i \tau_i) \xi \end{aligned}$$

where  $n_i$  is the unit vector in the direction of  $\mathbf{x}$ ,  $\tau_i$  are the three Pauli matrices,  $\xi$  is a constant unit doublet and  $a_0, a_1, \alpha, \beta, \mu_I, \nu_I$  are functions of  $r$  to be determined by the energy extremization. It is convenient to choose the gauge  $a_0 = 0$ . One then solves Gauss' constraint to obtain  $a_1 = 0$ <sup>7</sup>.

We rescale distances and fields by the appropriate powers of  $m_W = g\sqrt{(v_1^2 + v_2^2)}/2$  and use the notation  $\phi_I = \mu_I + i \nu_I \equiv F_I e^{i\Theta_I}$  to write for the energy functional:

$$\begin{aligned} E &= \frac{M_W}{\alpha_W} \frac{2}{1+t^2} \int dr \left\{ \frac{1+t^2}{2} (\alpha'^2 + \beta'^2) + r^2 (F_1'^2 + F_2'^2 + F_1^2 \Theta_1'^2 + F_2^2 \Theta_2'^2) \right. \\ &\quad \left. + \frac{1+t^2}{4r^2} (\alpha^2 + \beta^2 - 1)^2 + \frac{1}{2} (\alpha^2 + \beta^2 + 1) (F_1^2 + F_2^2) \right\} \end{aligned}$$

<sup>d</sup>The work in this section was done in collaboration with Dr. P. Tinyakov and is described in detail in reference [31].

<sup>e</sup>Though admittedly premature, some other physical properties of such would-be particles are fun to contemplate: being classically stable they could easily have cosmological life times. They would have a mass in the  $\sim 10 \text{ TeV}$  region, zero charge and dipole moments in their ground state, and geometrical interaction cross sections of order  $1/m_W^2$ . Assuming maximum production at the electroweak phase transition, a rough estimate of their present abundance shows that they could be candidates for cold dark matter in the universe.

$$\begin{aligned}
& -\alpha[F_1^2 \cos(2\Theta_1) + F_2^2 \cos(2\Theta_2)] - \beta[F_1^2 \sin(2\Theta_1) + F_2^2 \sin(2\Theta_2)] \\
& + \frac{\kappa_1^2}{4}r^2(F_1^2 - 1)^2 + \frac{\kappa_2^2}{4t^2}r^2(F_2^2 - t^2)^2\}, \tag{2}
\end{aligned}$$

where  $t \equiv v_2/v_1$  and  $\kappa_I \equiv m_{H_I}/m_W$ . The radial coordinate in particular in the above expression is measured in units of  $m_W^{-1}$ .

We require finiteness of the energy of the solution and use the global symmetries of the energy functional<sup>f</sup> to specify the boundary conditions at  $\infty$  and write them in the form:

$$\alpha \rightarrow 1, \quad \beta \rightarrow 0, \quad F_1 \rightarrow 1, \quad F_2 \rightarrow t, \quad \Theta_a \rightarrow 0$$

Correspondingly, at  $r = 0$  energy finiteness implies

$$\alpha^2 + \beta^2 \rightarrow 1.$$

while the field equations obtained by varying (3) with respect to  $\Theta_I$ , together with the requirement of smoothness of the solution lead to the dynamical conditions:  $2\Theta_a - \psi = 0 \bmod 2\pi$ , where  $\psi$  is the phase of the field  $\chi \equiv \alpha + i\beta$ . Thus all solutions satisfy  $\Theta_1 - \Theta_2 = \pi N$  at the origin.

We have looked for solutions in a wide region of the  $\{t, \kappa_1, \kappa_2\}$  parameter space. We refer the reader to [31] for the detailed presentation of the numerical method we used and of the entire solution-zoo we have obtained so far. In the present note, I will describe briefly some of the solutions we found for various values of  $\kappa$  on the line  $\{t = 1, \kappa_1 = \kappa_2 \equiv \kappa\}$ , together with their main characteristics. As we vary  $\kappa$  the energy landscape changes smoothly and consequently, one generically expects a continuous change in the number and properties of its extrema and saddles. Figure 1 below captures these changes in part of the energy landscape and shows some of the solutions of the model at hand.

The graph should be thought of as an "artist's conception" of a part of the tree of solutions with  $N = 0$  and  $|N| = 1$  which has emerged so far in the two-Higgs model under study. Each branch of the "tree" corresponds to a particular type of solution. As we move along a branch by varying  $\kappa$ , continuous quantities like the detailed form of the solution, its energy as well as the value(s) of the imaginary eigen-frequencies of the small oscillations around it change smoothly, while integer-valued ones such as the index  $N$ , the number of unstable negative curvature modes (shown on the branch) and the number of nodes of the fields, remain invariant. We have suppressed the branches corresponding to the CP-conjugates of the solutions shown. They may be imagined as the mirror images with respect to the  $\kappa$ -axis of the branches above it.

For  $\kappa$  between 0.0 and 5.5 the only non-trivial solution is the sphaleron embedded in the two-Higgs model; it has  $\Theta_1 = \Theta_2 = 0$ ,  $\beta = 0$  and one negative mode. For  $\kappa$  about 5.5 we reach the bifurcation point  $B_0$ , the sphaleron develops a second negative mode and at the same time two new solutions, namely the branch  $A_1$  and its CP conjugate appear with  $N = 1$  and  $N = -1$  respectively. The phases  $\Theta_1$  and  $\Theta_2$  and the complex field  $\chi$  of the solution corresponding to the branch  $A_1$  have the behaviour we call of "type-A" and show in figure 2.

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<sup>f</sup>The energy functional is invariant under the simultaneous rotation of the complex fields  $\phi_1, \phi_2$  and  $\alpha + i\beta$  by an angle  $\omega, \omega$  and  $2\omega$  respectively, as well as under independent rotations of  $\phi_I$  by  $\pi$ .

We have checked that the branch  $A_1$  extends without a new bifurcation up to  $\kappa = 200$ . As for the sphaleron, once we get to  $\kappa \simeq 12.0$  we encounter the bifurcation  $B_1$ . The sphaleron acquires a third unstable mode and two new solutions (branch  $S_2$  and its CP-conjugate) emerge. They have  $N = 0$  and phase behaviour of "type-S" shown above. And so on for the rest of the tree.

Incidentally, using the number of negative modes  $N_{\text{neg. modes}}$  of the solutions shown in figure 1 one may explicitly verify the sum rule:  $\sum_{\text{solutions}} (-)^{N_{\text{neg. modes}}} = \text{constant}$  independent of  $\kappa$ , for the whole range of values we considered.

## 4. Discussion

It is clear that the two-Higgs system discussed here possesses a variety of unstable solutions, whose richness increases with the Higgs masses. These, like the ordinary sphaleron, are in principle interesting in their own right especially in connection with the baryon number generation at the electroweak phase transition. There are important differences<sup>31</sup> in the values of the energies and of the negative modes of the above solutions compared to analogous ones of the minimal standard model. For instance, the value of the negative curvature along the  $A_1$  branch is about half of what it is in the corresponding least unstable deformed sphaleron of the one-Higgs model. These differences affect directly the prediction for the baryon number in the Universe, and at the same time they offer support to our general arguments that an extended Higgs sector improves the stability of the winding solitons under discussion.

Despite of this indirect evidence though, we have not as yet been able to find a stable soliton. Such a solution might for example arise at a bifurcation point like X of figure 1 and either merge with another branch of the tree at some new bifurcation or continue, as is the case with  $A_1$ , up to  $\kappa = \infty$ . But since our numerical method requires an initial guess for the solution which has to lie inside the basin of attraction of the corresponding local minimum of the energy functional, our failure to find a stable solution might just be due to the bad starting configurations we tried so far. Further theoretical analysis is required to guide our numerical search.

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